

CMB master
UE pro

Clémence FOURNIÉ

3rd year PhD student
Former CMB master student

01/12/2025



LABORATOIRE
MAP5



Université
Paris Cité

After CMB master

Thanks to “math-bio-
sante” diffusion list

PhD subject:

Cell Trajectory Inference based on Schrödinger problem and a mechanistic stochastic gene expression model

Directors : Fabien Crauste (CNRS, MAP5, Paris)
and Olivier Gandrillon (ENS Lyon)

In collaboration with Elias Ventre (INRIA Marseille), Ulysse Herbach (CNRS, Inria, IECL) et Aymeric Baradat (ICJ Lyon)

Financed by CNRS

1. Mechanistic stochastic gene expression model

2. Biological experience

3. Schrödinger problem

4. Simulated data

5. Reference process

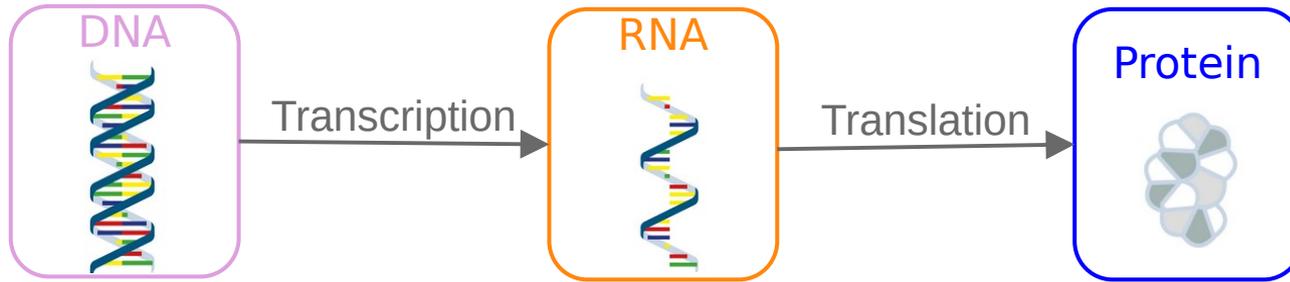
6. Result

7. Discussion and next step

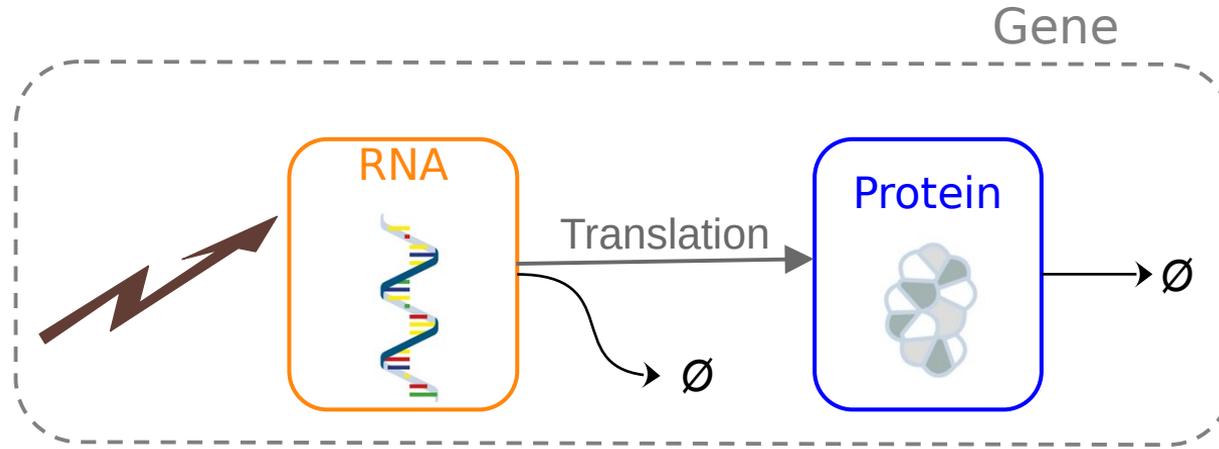
Slides available:

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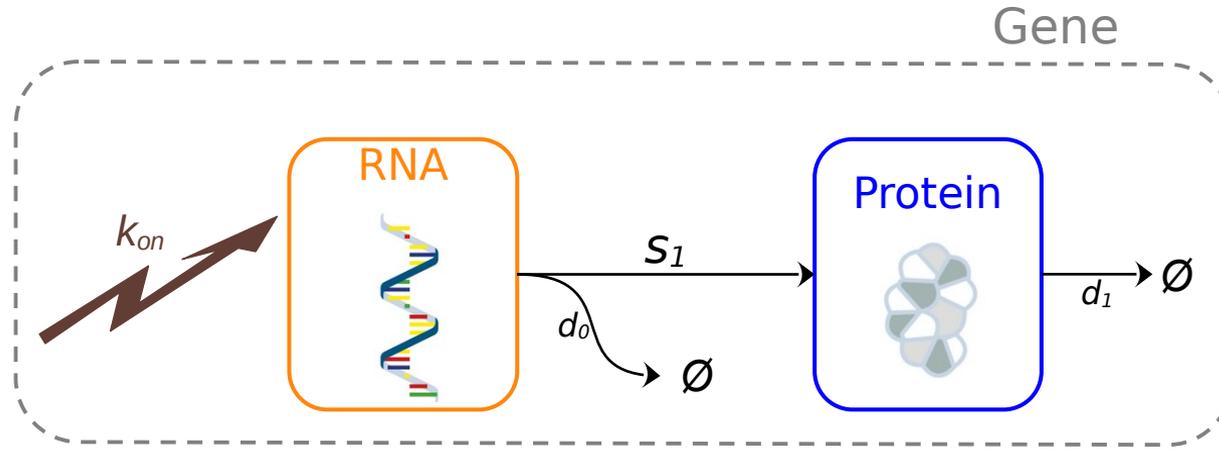
Gene



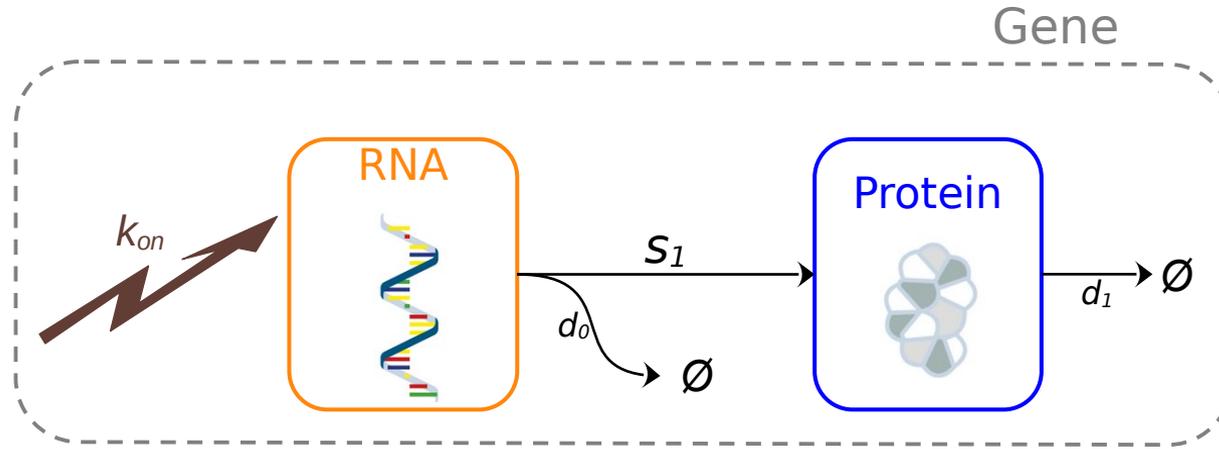
Gene



Gene



Gene – Bursty model

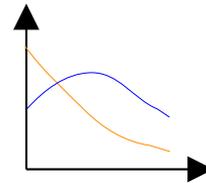


k_{on}

$$\begin{cases} \dot{M}(t) = -d_0 M(t), \\ \dot{P}(t) = s_1 M(t) - d_1 P(t). \end{cases}$$

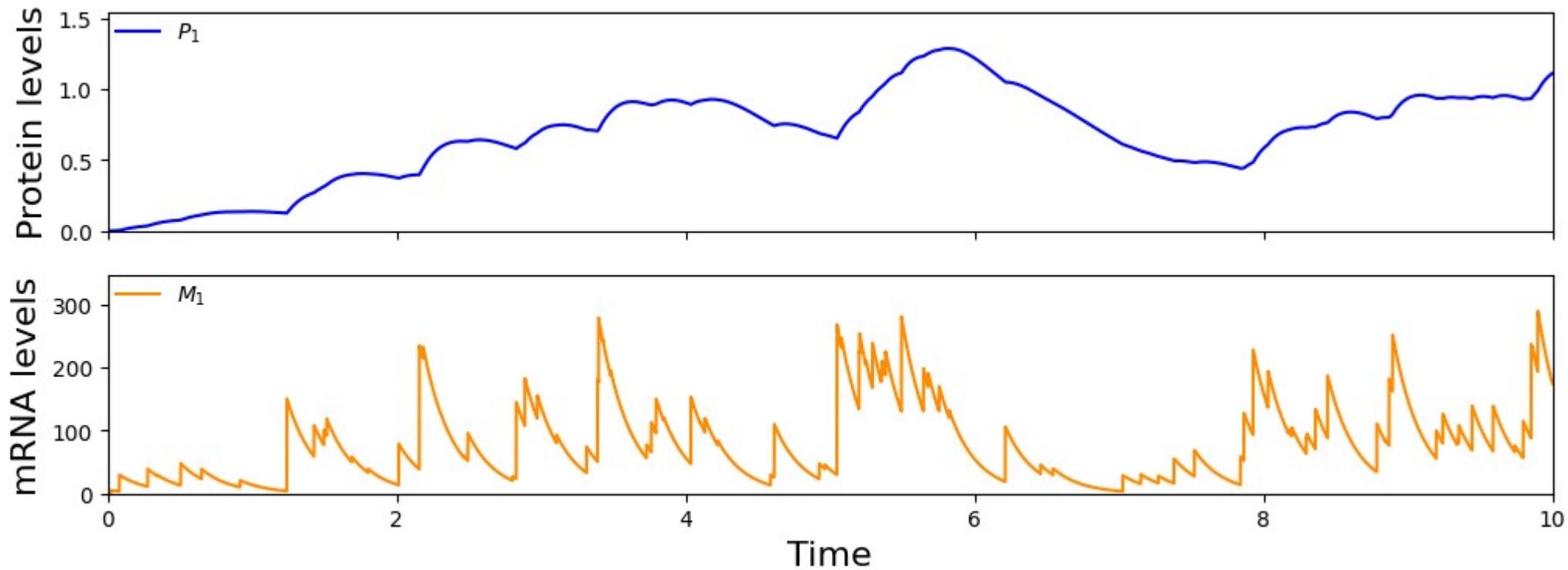


$M(t)$



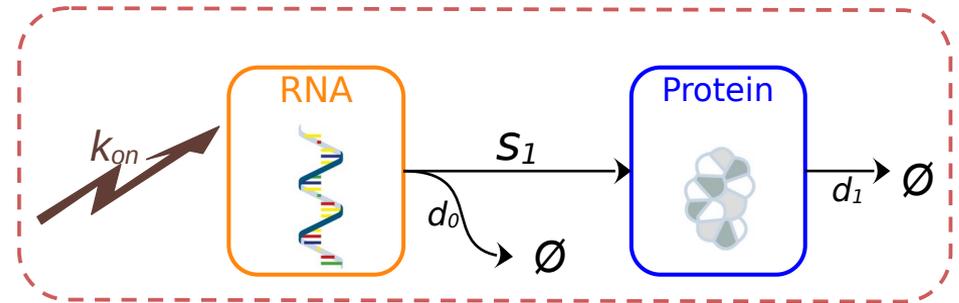
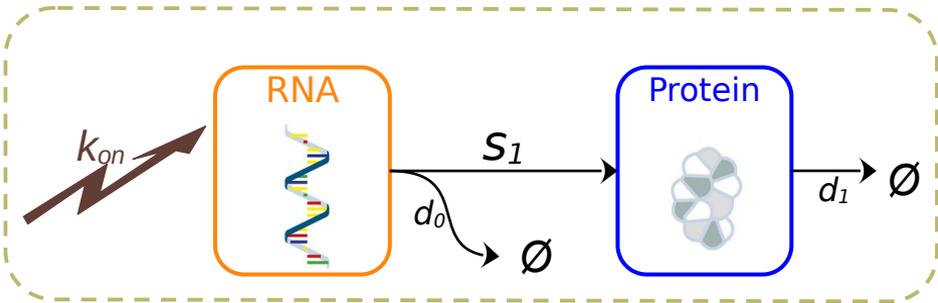
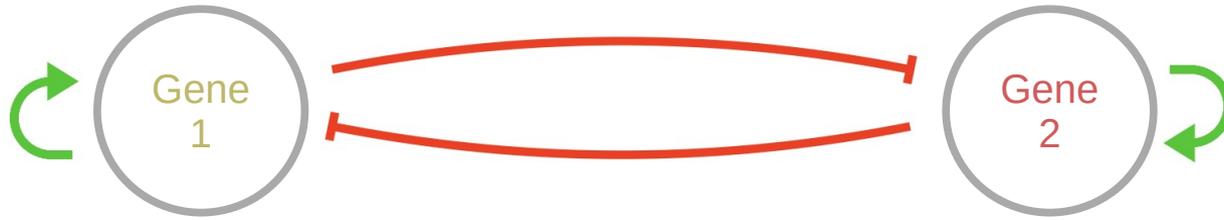
(Herbach et al. 2022)

Bursty model

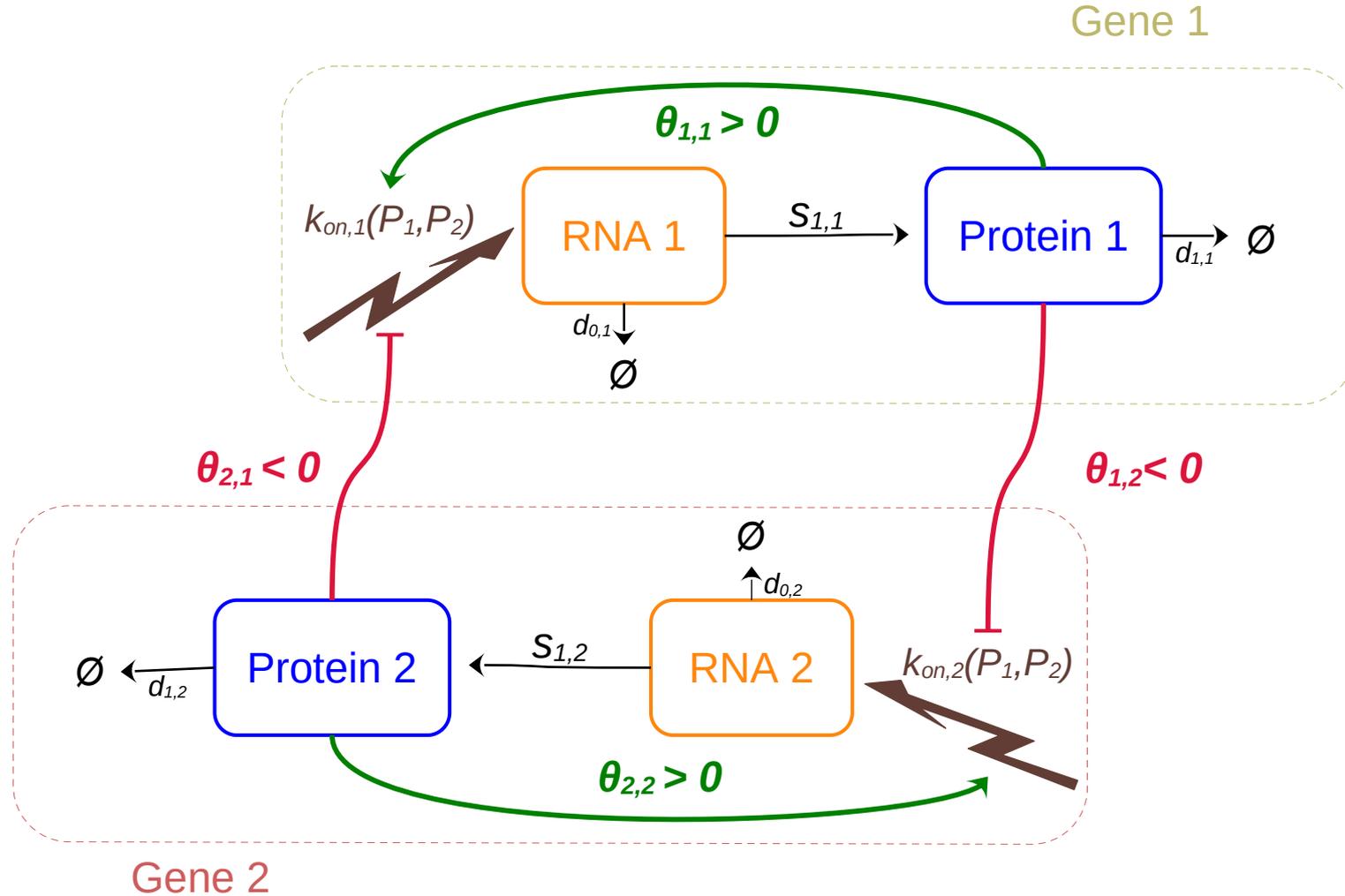


Simulated with HARISSA (Herbach, 2023).

Gene Regulatory Network

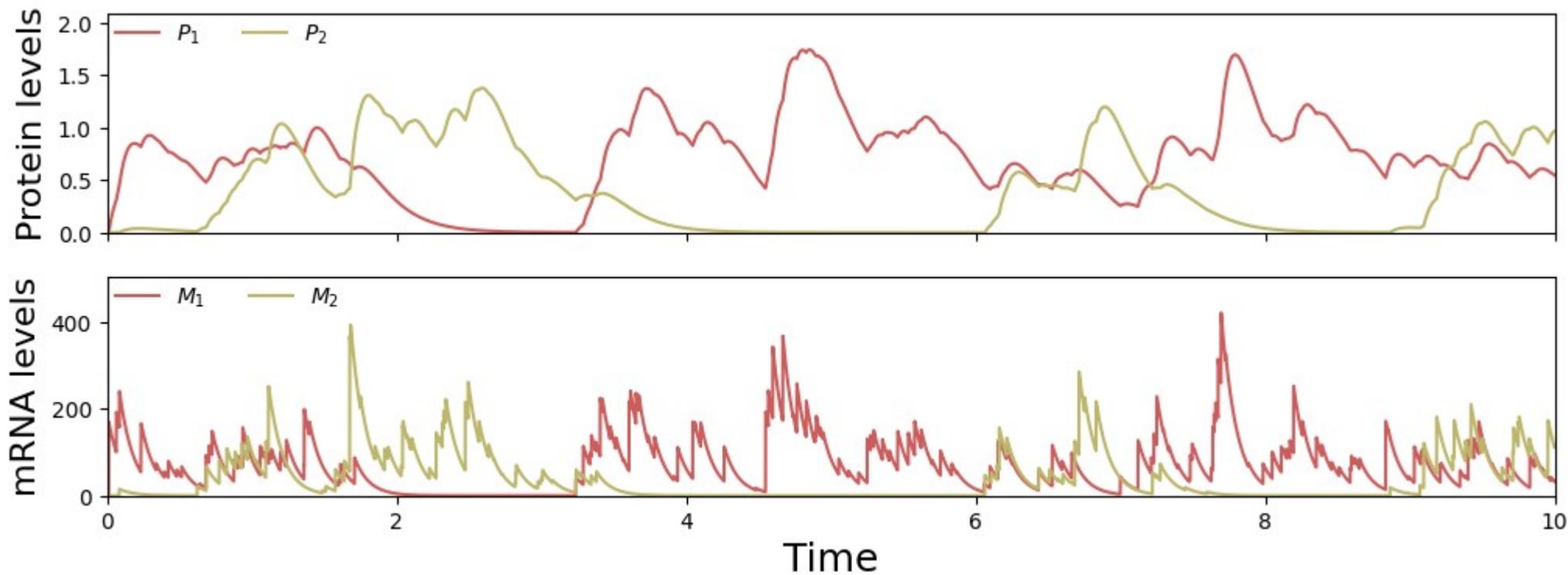


Bursty Model Toggle Switch



Bursty Model

Toggle Switch



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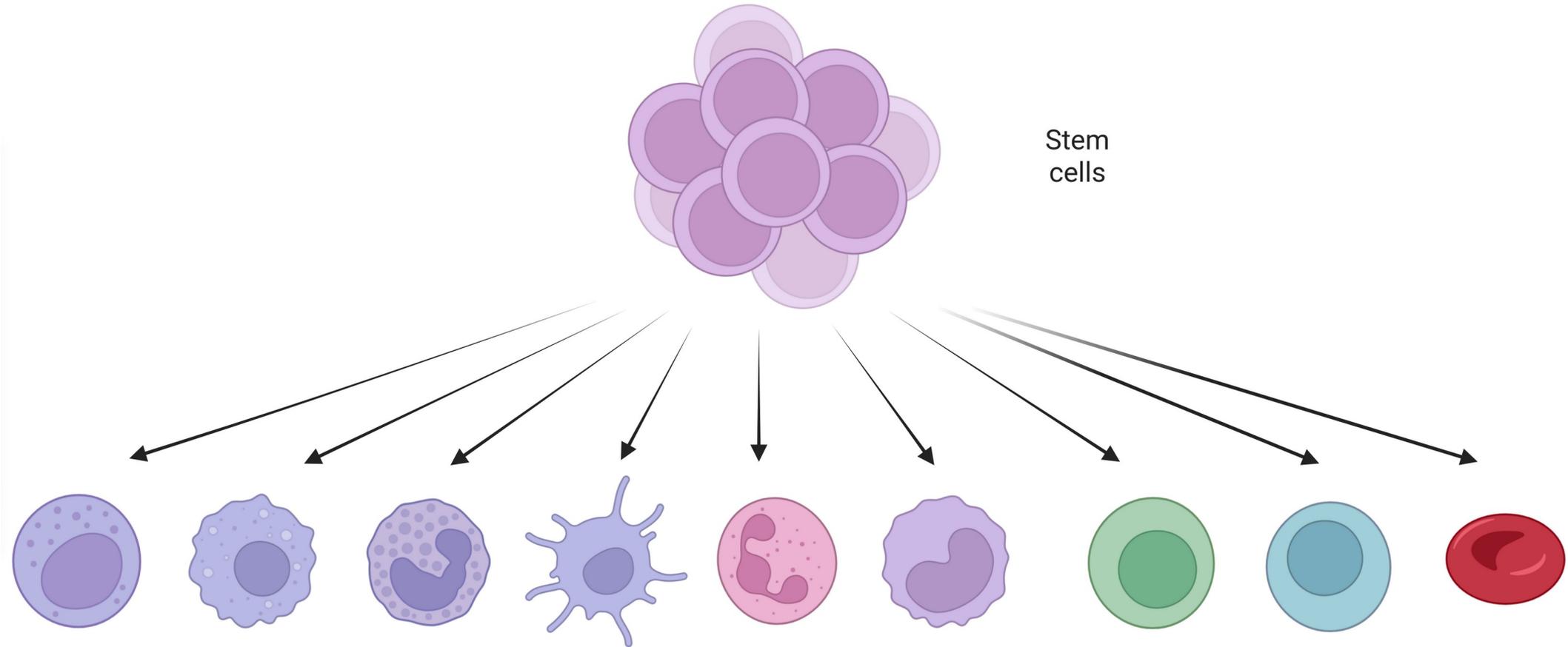
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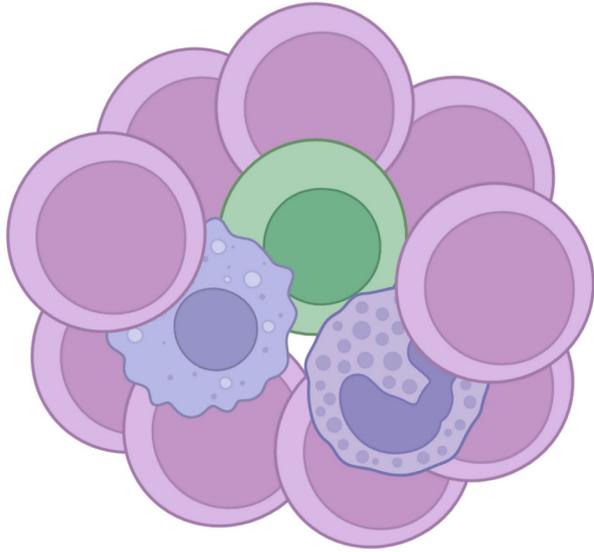
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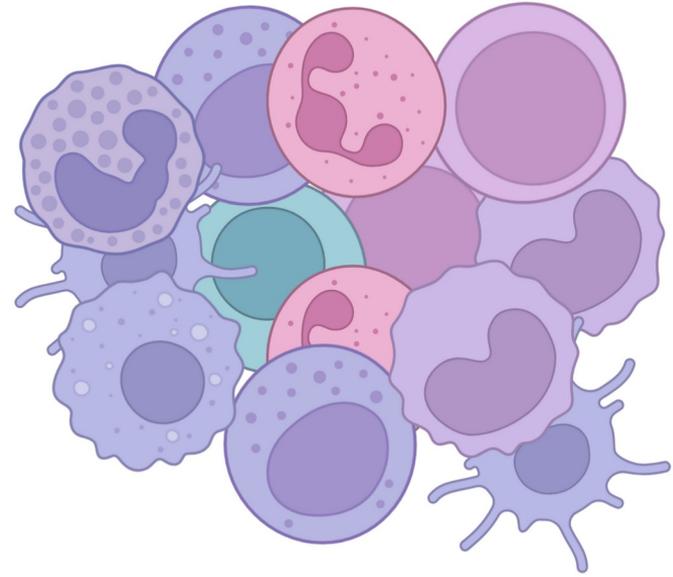
Cellular differentiation



Cellular differentiation

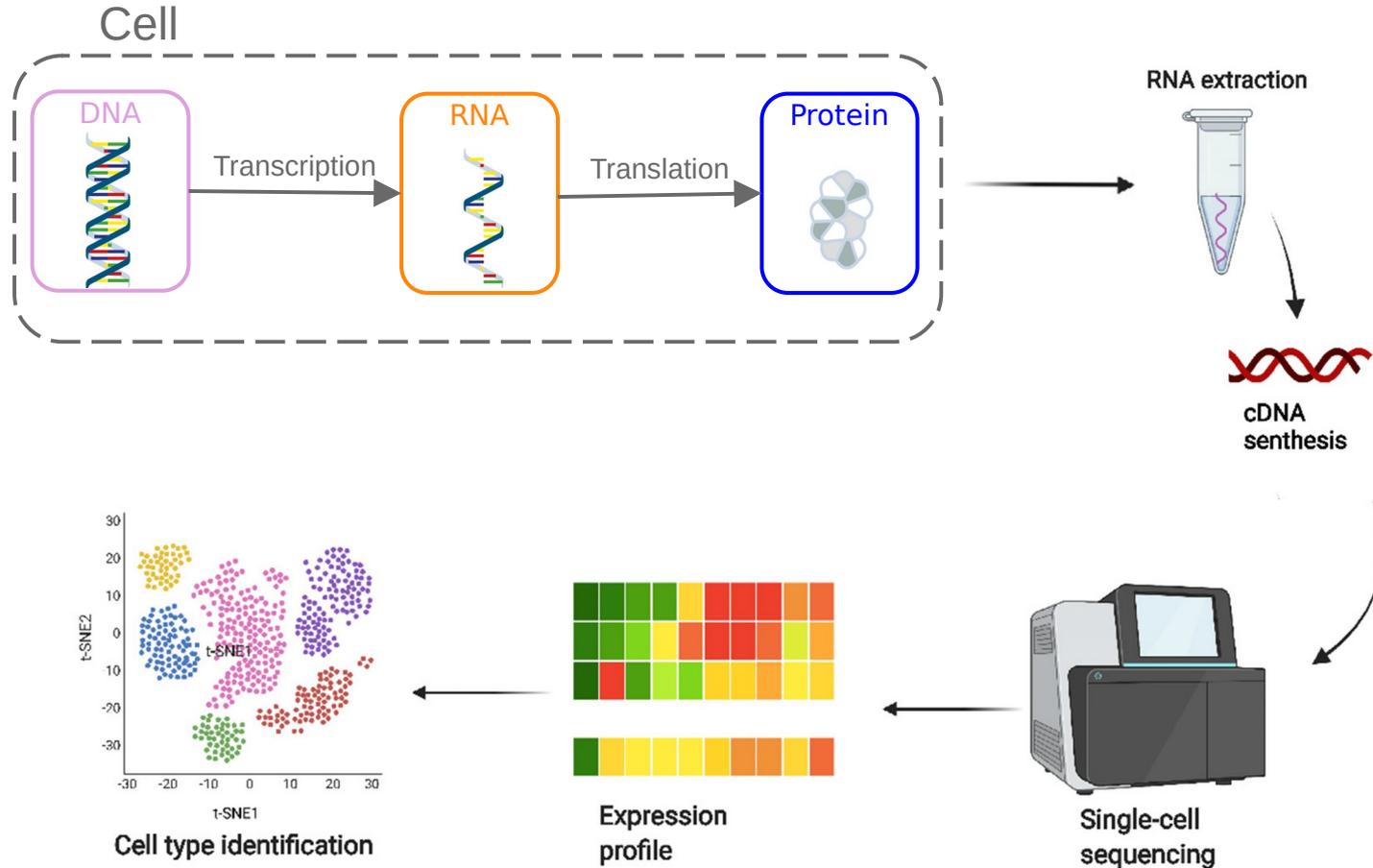


t_1



t_2

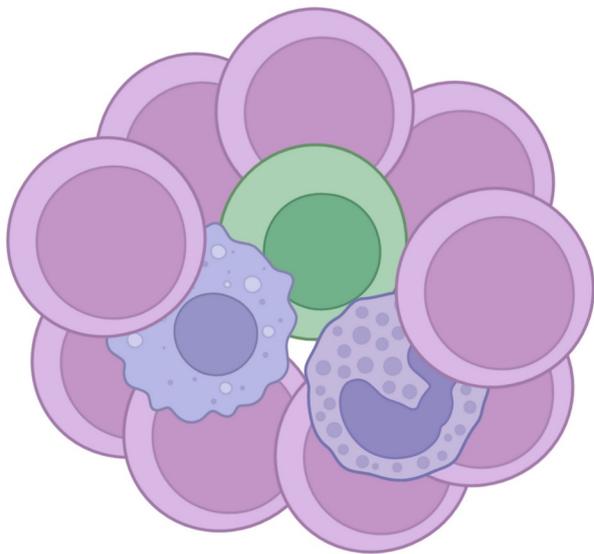
Single Cell RNA seq



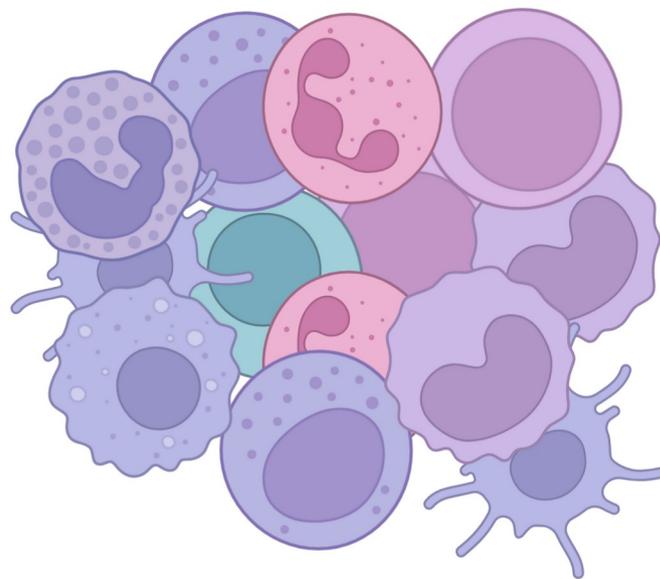
(Adapted from Bawa et al. 2022)

Context

Experiment 1



Experiment 2

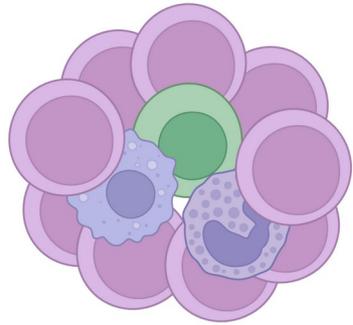


t_1

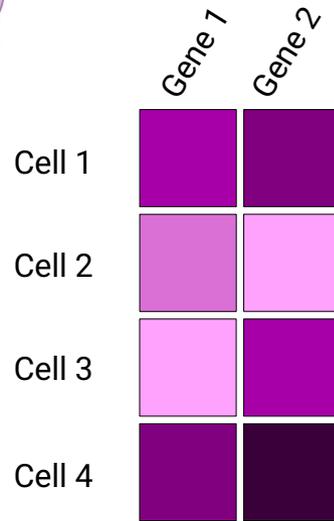
t_2



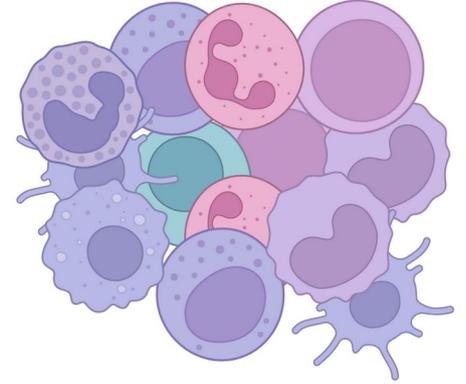
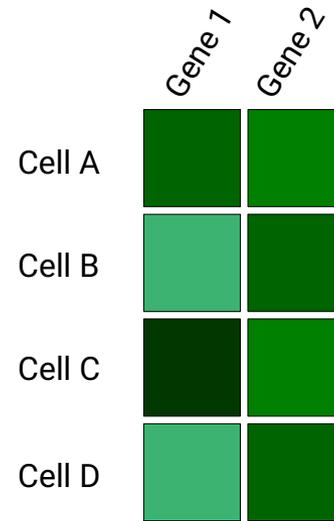
Context



Experiment 1



Experiment 2

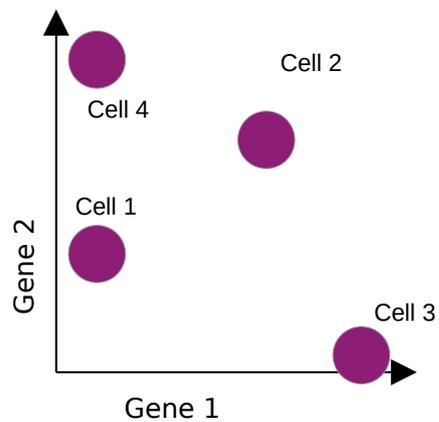


t_1

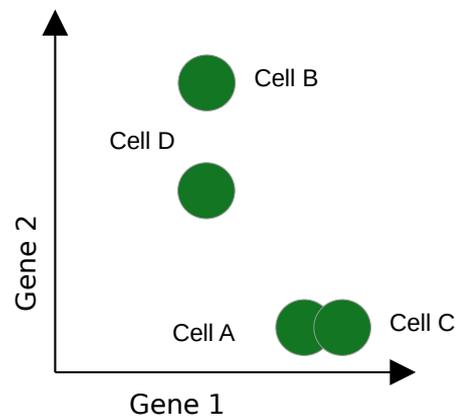
t_2

Context

Experiment 1



Experiment 2



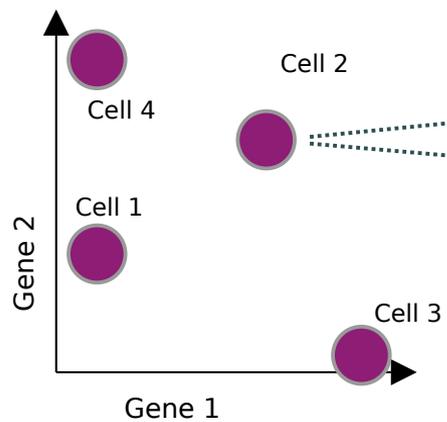
t_1

t_2

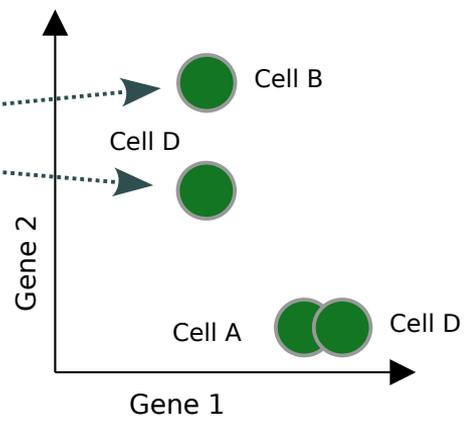


Context

Experiment 1



Experiment 2



?

?

t_1

t_2



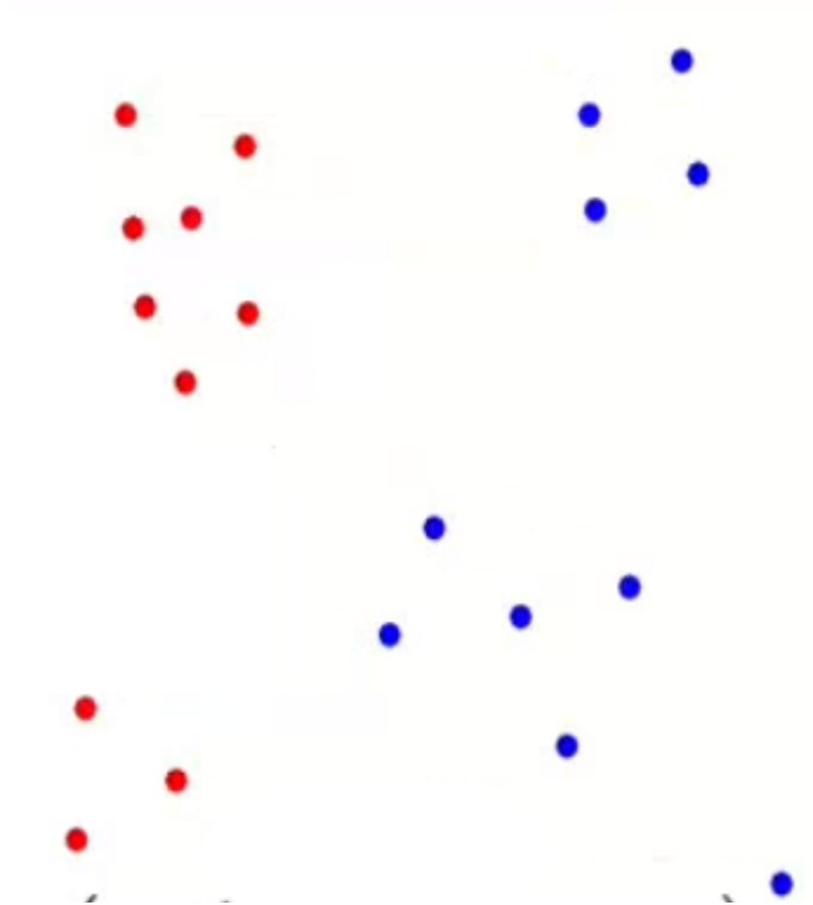
1. Mechanistic stochastic gene expression model
2. Biological experience
- 3. Schrödinger problem**
4. Simulated data
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Schrödinger Problem

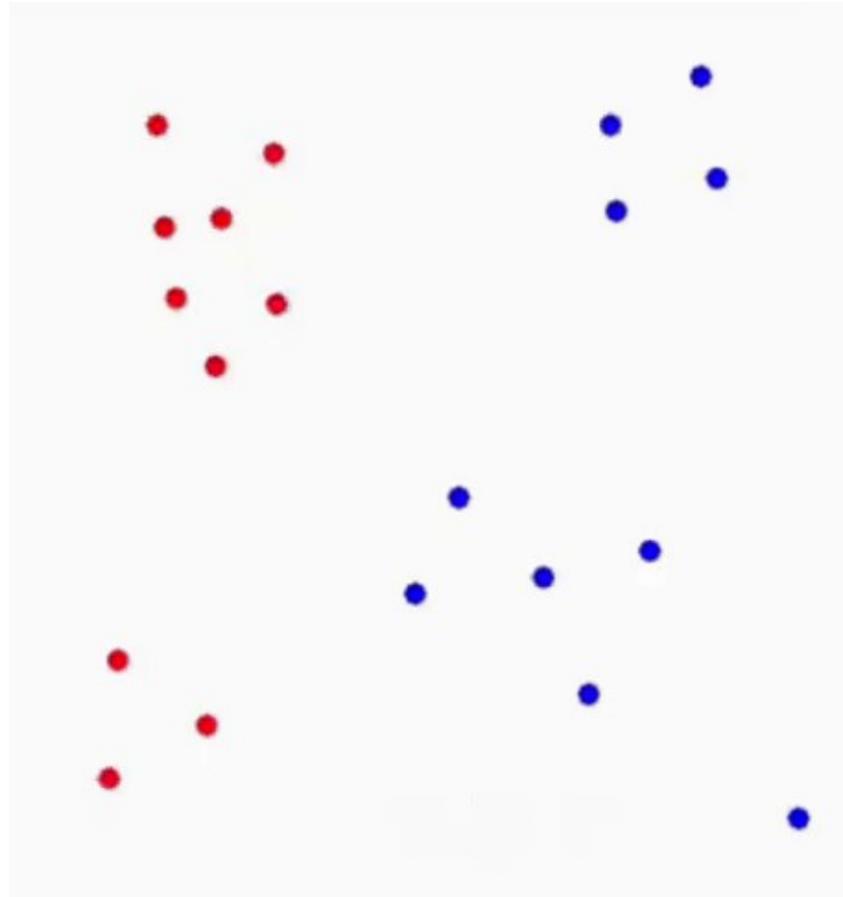
μ



ν

Schrödinger Problem

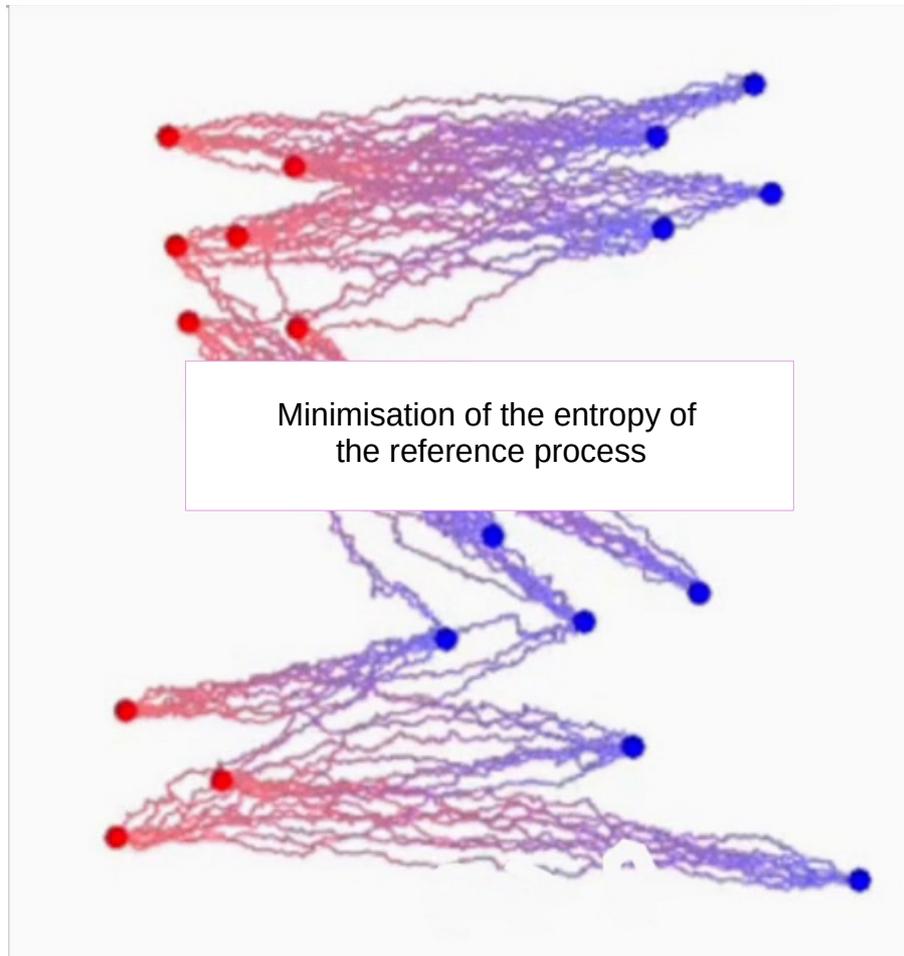
μ



ν

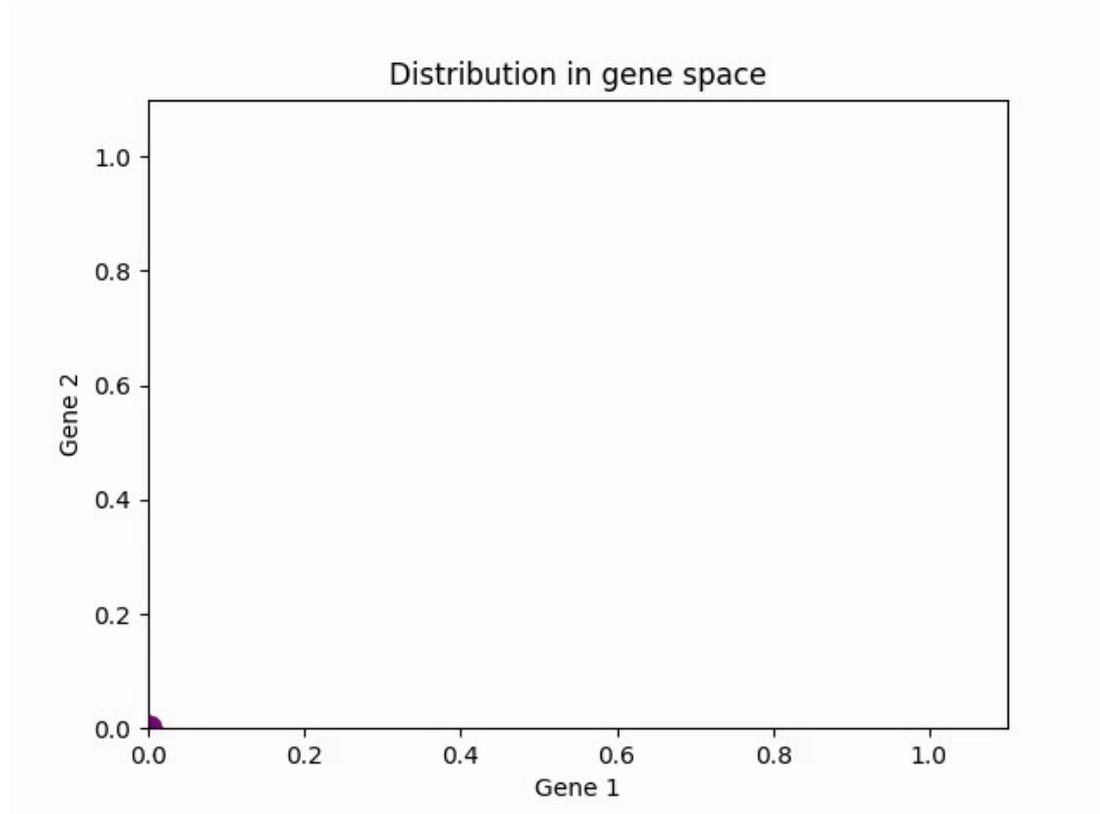
Schrödinger Problem

μ

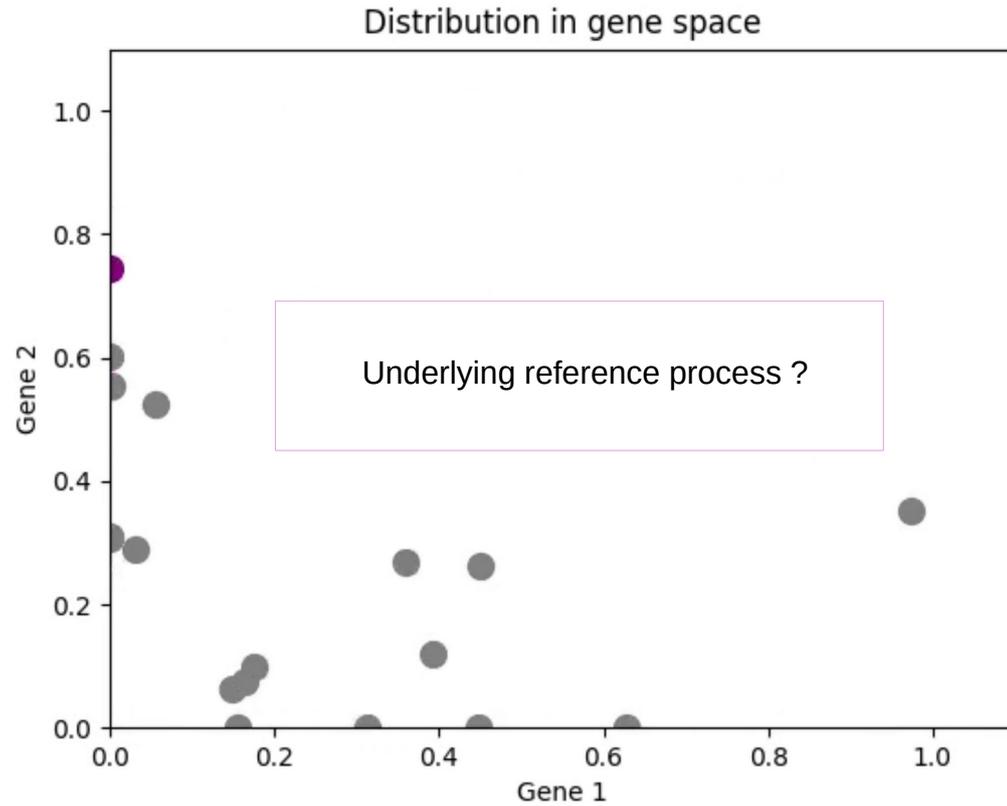


ν

Cells trajectories in 2D



Cells trajectories in 2D

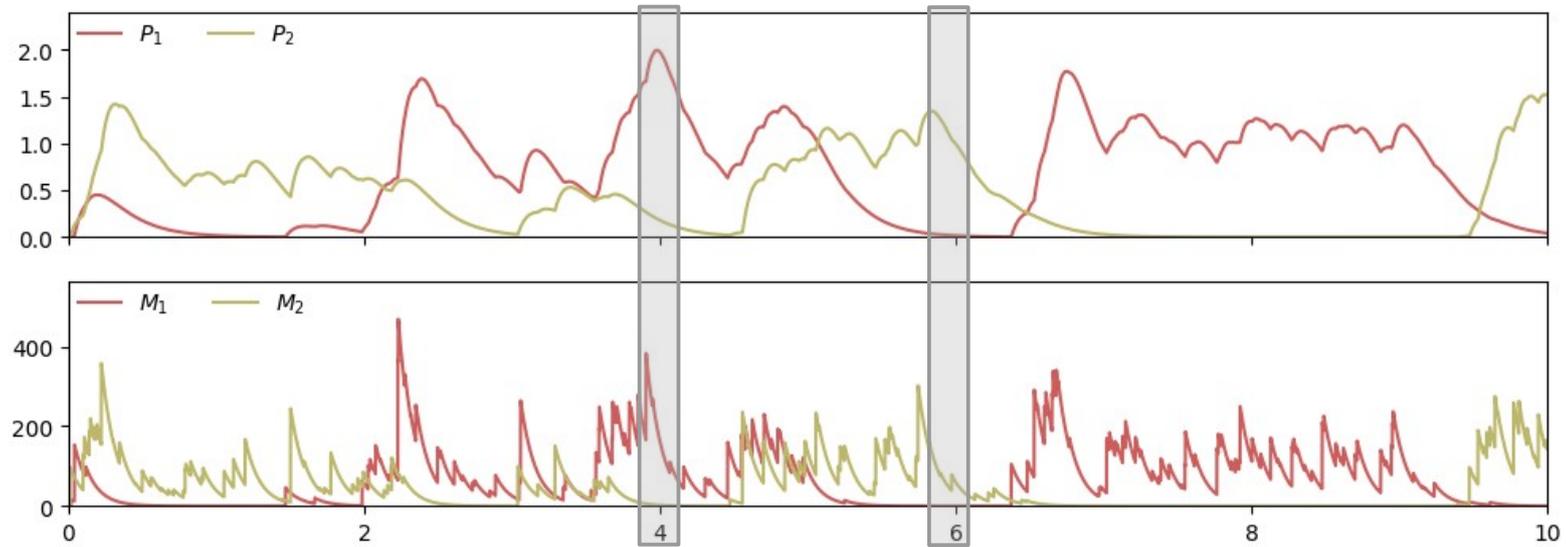


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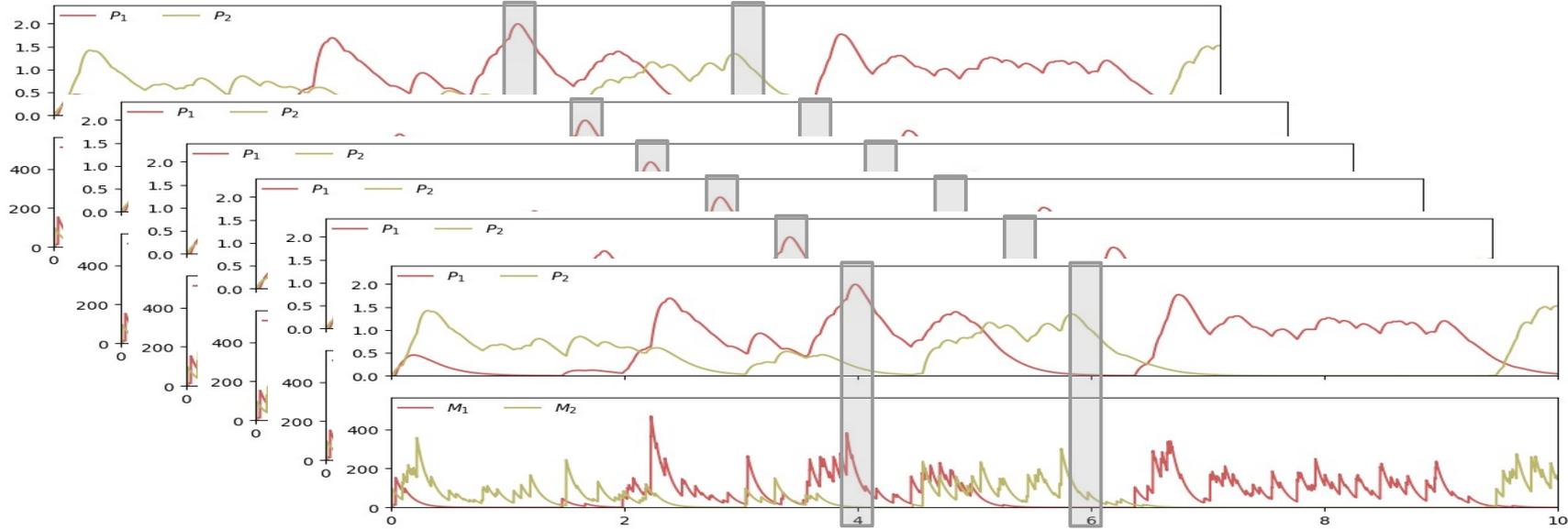
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Methods: Simulated data



Methods: Simulated data

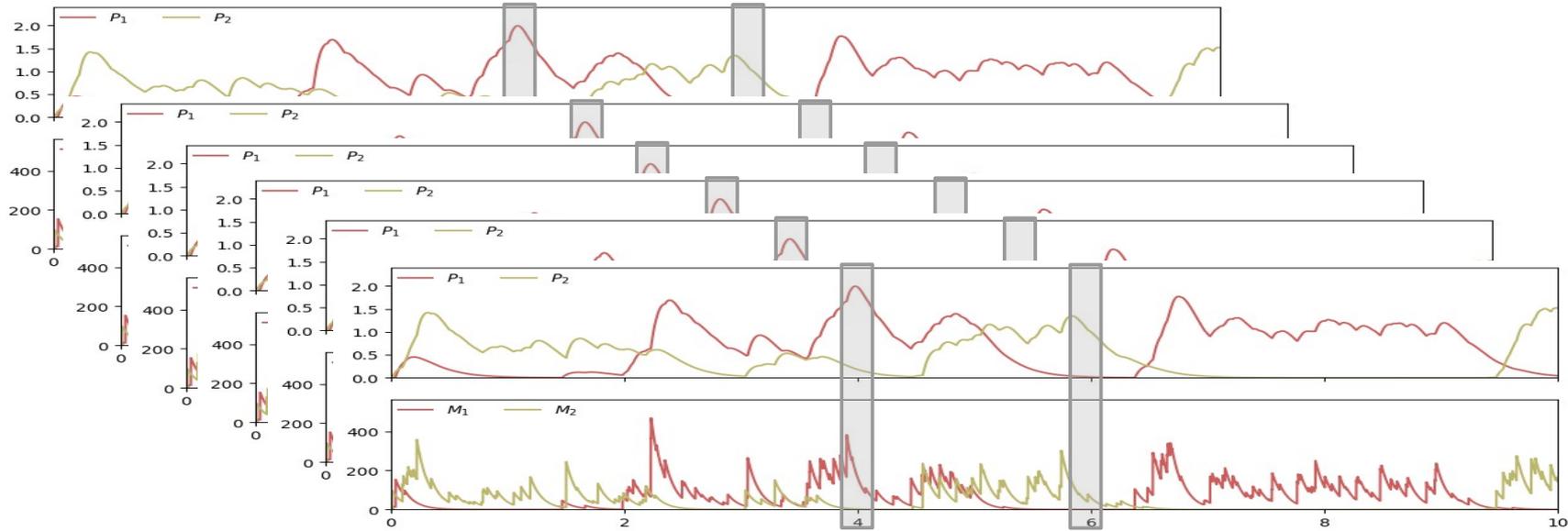


N time

$$\begin{aligned} & \mathbf{t}_1 \\ \mathbf{x}_1 &= [(P_1, P_2), (M_1, M_2)] \\ & \cdot \\ & \cdot \\ \mathbf{x}_N &= [(P_1, P_2), (M_1, M_2)] \end{aligned}$$

$$\begin{aligned} & \mathbf{t}_2 \\ \mathbf{y}_1 &= [(P_1, P_2), (M_1, M_2)] \\ & \cdot \\ & \cdot \\ \mathbf{y}_N &= [(P_1, P_2), (M_1, M_2)] \end{aligned}$$

Methods: Simulated data



N time

μ

$$\begin{array}{l}
 \mathbf{x}_1 = \begin{bmatrix} (P_1, P_2) \\ (M_1, M_2) \end{bmatrix} \\
 \vdots \\
 \mathbf{x}_N = \begin{bmatrix} (P_1, P_2) \\ (M_1, M_2) \end{bmatrix}
 \end{array}$$

t_2

$$\begin{array}{l}
 \mathbf{y}_1 = \begin{bmatrix} (P_1, P_2) \\ (M_1, M_2) \end{bmatrix} \\
 \vdots \\
 \mathbf{y}_N = \begin{bmatrix} (P_1, P_2) \\ (M_1, M_2) \end{bmatrix}
 \end{array}$$

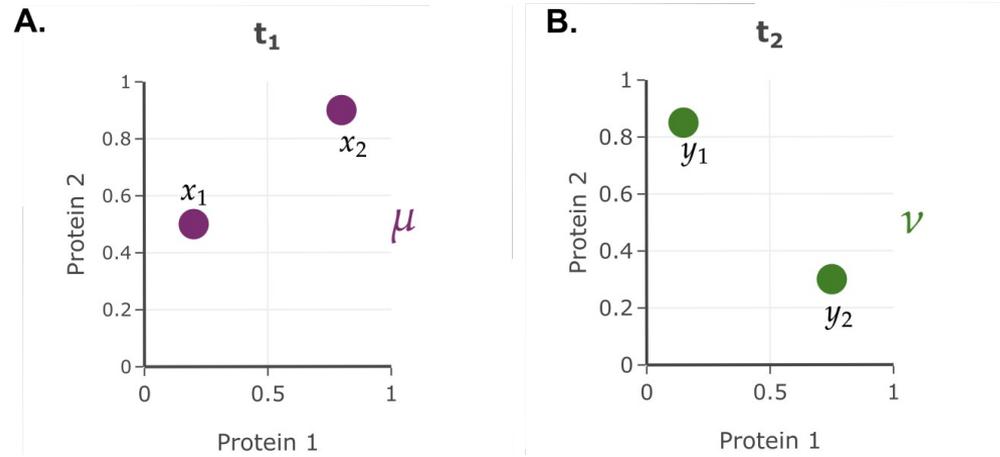
ν

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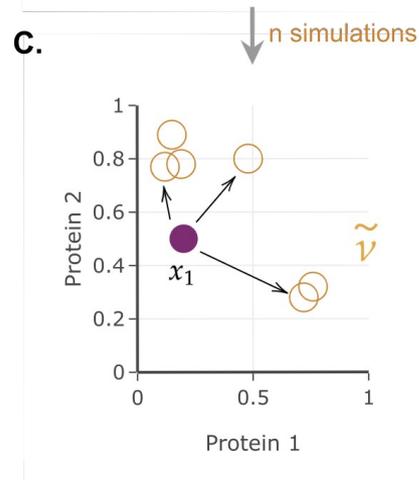
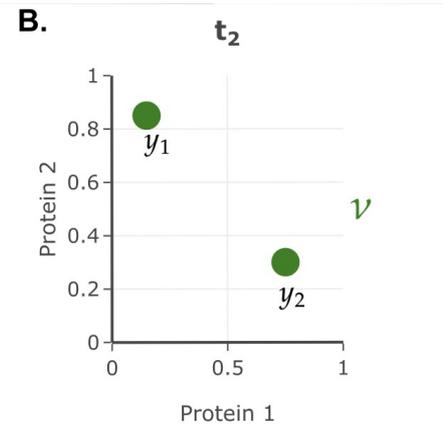
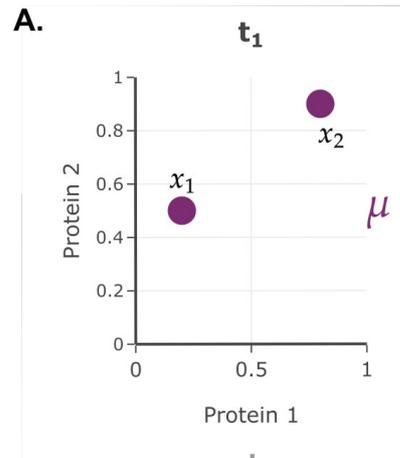
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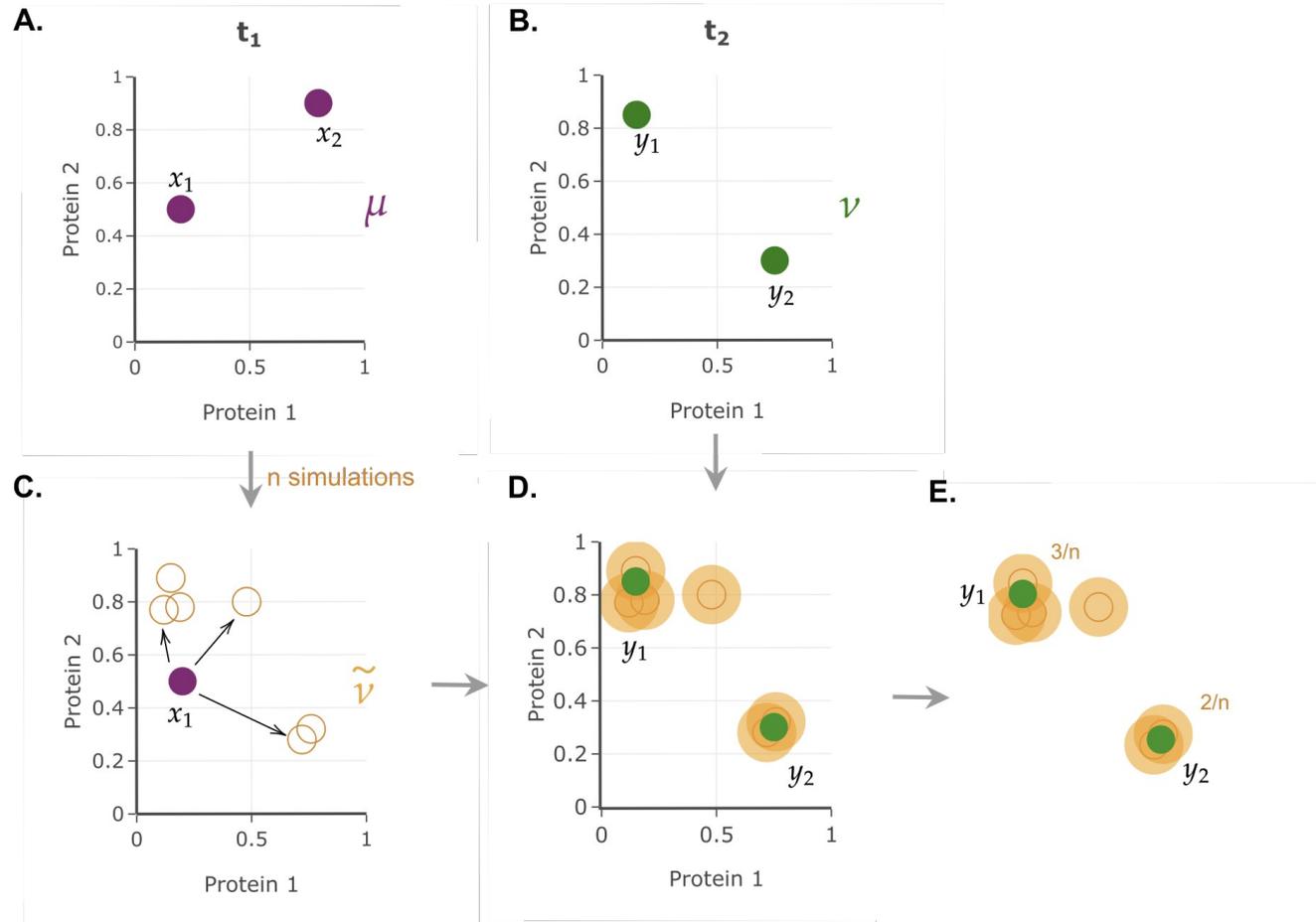
Methods: Approximation of the reference process



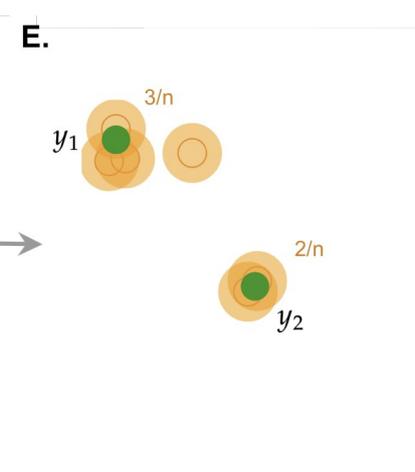
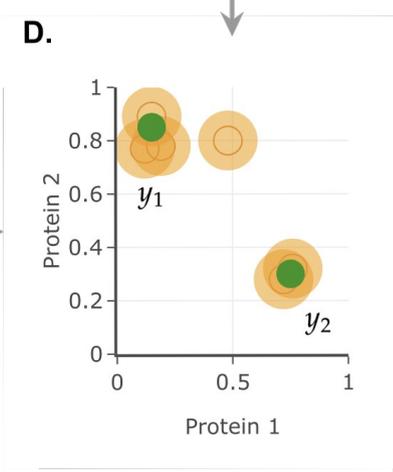
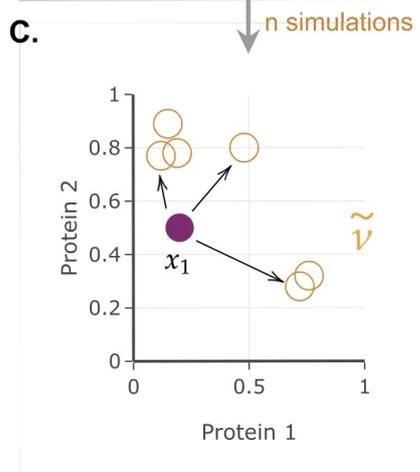
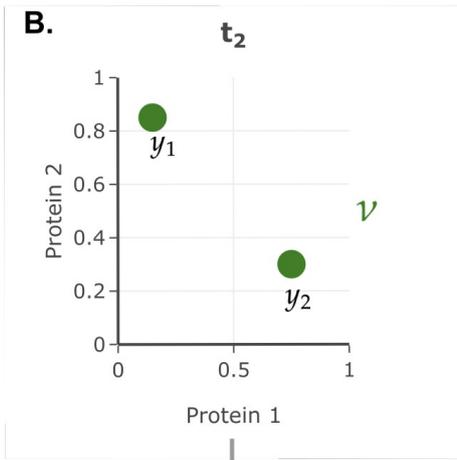
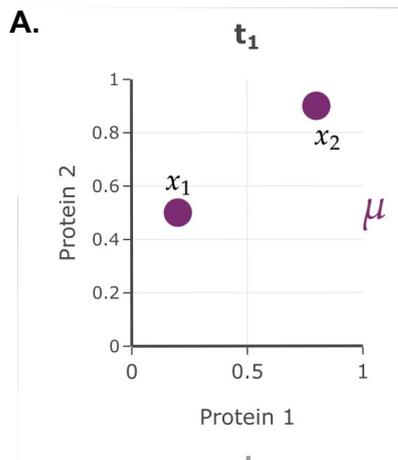
Methods: Approximation of the reference process



Methods: Approximation of the reference process



Methods: Approximation of the reference process



$$= B_{\text{ref}}$$

Methods: Schrödinger Problem

$$\boxed{B_{\text{sch}}} := \arg \min_{Q \in \mathcal{P}(X \times Y)} \{H(Q|B_{\text{ref}}); Q(t_1) = u, Q(t_2) = v\}$$

$$\text{With } H(Q|R) = \sum_{ij} Q_{ij} \log \left(\frac{Q_{ij}}{R_{ij}} \right)$$

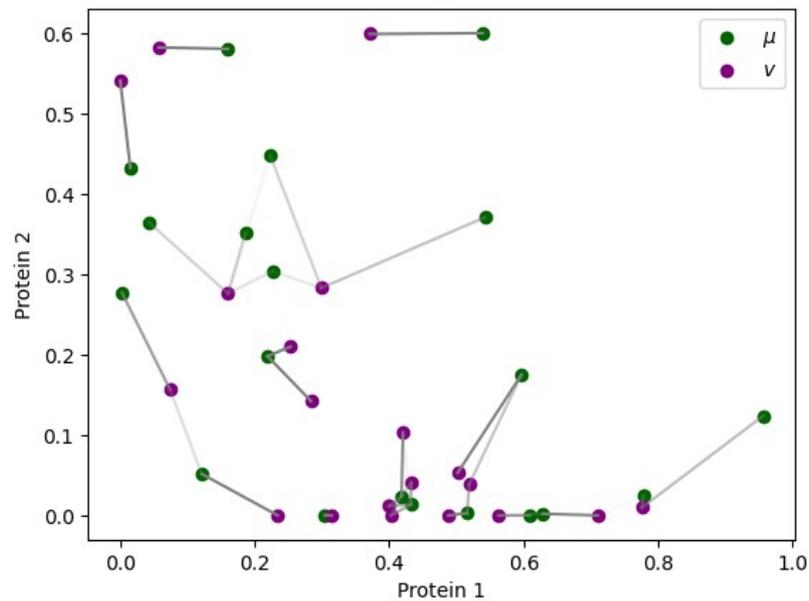
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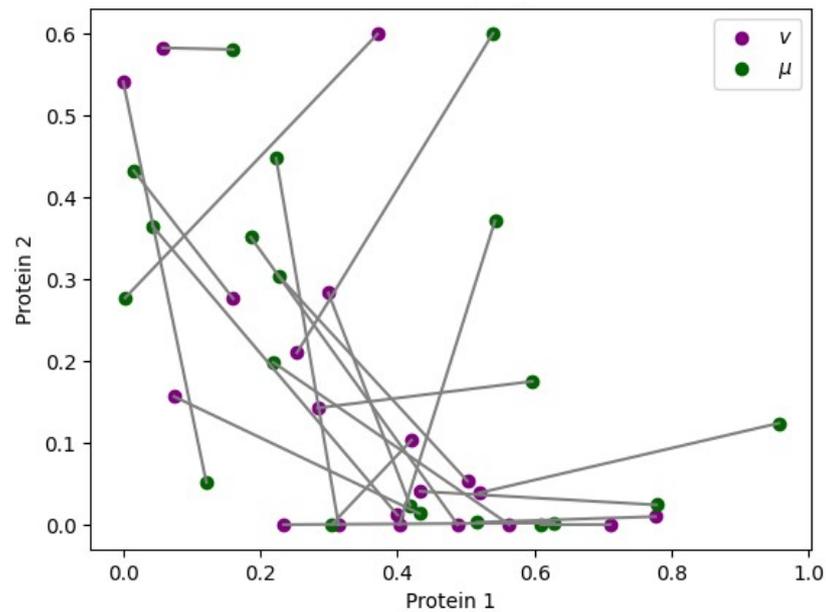
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Result

Coupling with diffusive process

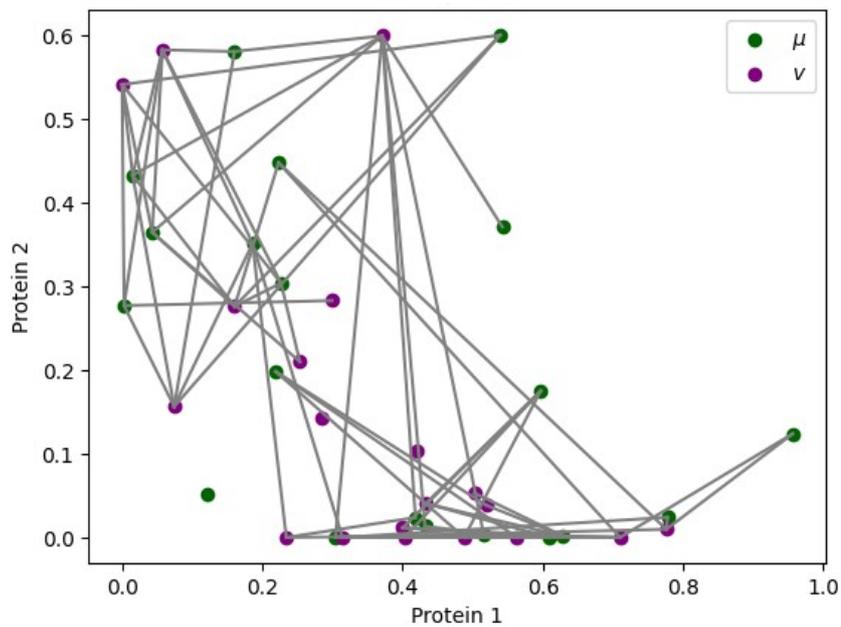


Real Coupling

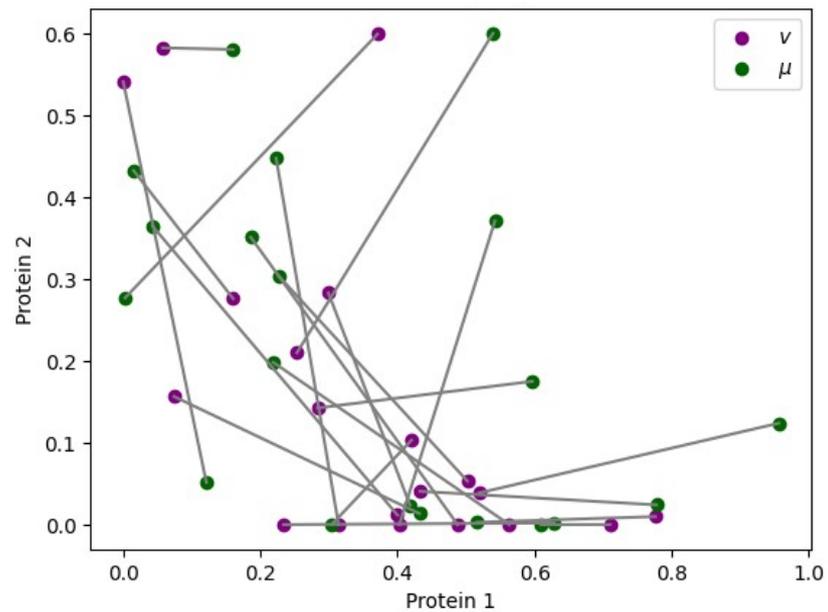


Result

Coupling with Bursty process



Real Coupling



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Discussion

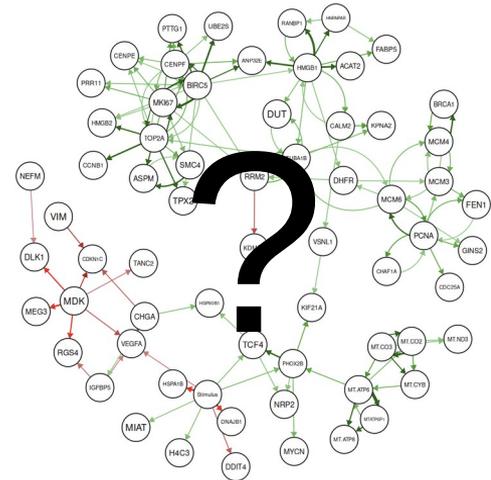
How to extend the model for real data ?

Discussion

How to extend the model for real data ?

RNA data ?

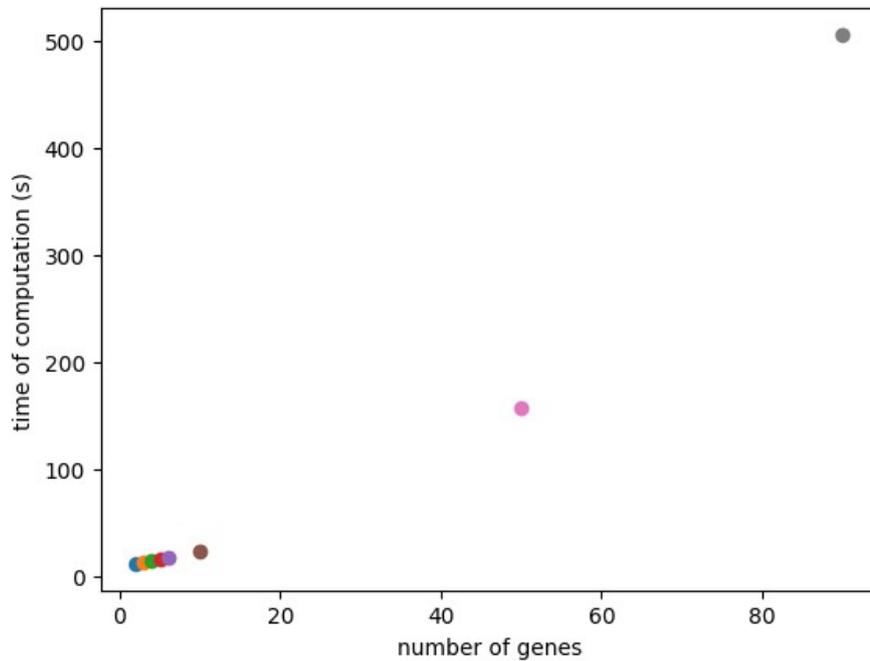
Computational time ?



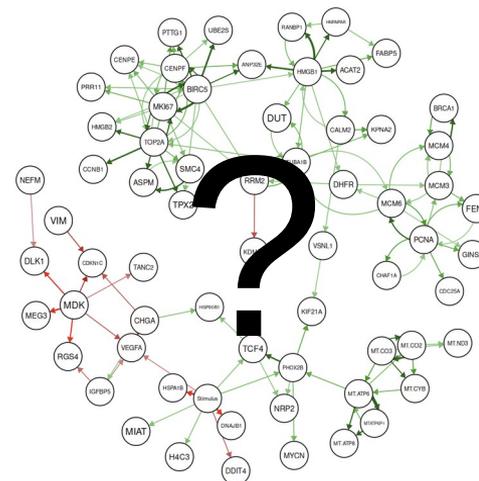
Discussion

How to extend the model for real data ?

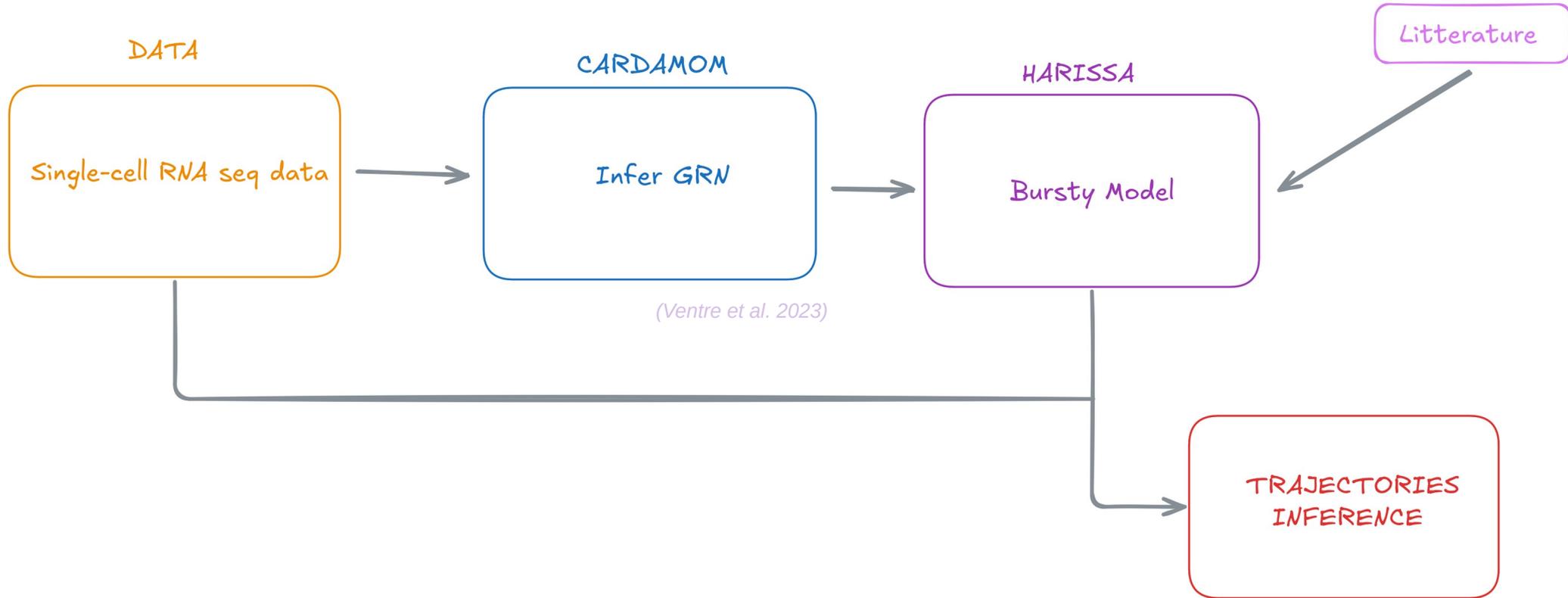
RNA data 



Computational time 



Discussion : Next steps



Thank you for your attention

Contact: clemence.fournie@math.cnrs.fr